Indian Statistical Institute, Bangalore Centre
B.Math III Year, First Semester
Semestral Examination
Differential Equations

Time: 3 Hours November 30, 2012 Instructor: C.R.E. Raja

Answer any five, each question carries 6 marks, total marks: 30

1. Solve $y' + P(x)y = Q(x)y^n$ where $n \geq 2$ and P, Q are continuous and apply it to solve $xy' + y = x^4y^3$.

2. Find the general solution of $x^2y'' - 2xy' + 2y = 0$ on a interval J not containing 0. Can one determine any solution of $x^2y'' - 2xy' + 2y = 0$ on any interval I. Justify your answer.

3. Find a particular solution of $y'' + 2y' + 5y = e^{-x} \sec 2x$ using the method of variation of parameters.

4. Let f and $\frac{\partial f}{\partial y}$ be continuous functions on $[a, b] \times \mathbb{R}$. If y_1 and y_2 are solutions of $y'(x) = f(x, y(x))$ on $[a, b]$, prove that $\{t \in (a, b) \mid y_1(t) = y_2(t)\}$ is (a, b) or $\emptyset$.

5. Find a solution u of $u_t = u_{xx}$ by SV-method that satisfies $\lim_{t\to\infty} u(x, t) = 0$.

6. Let Ω be a bounded connected open set in $\mathbb{R}^2$ and $u \in C^2(\Omega)$ be $u_{xx} + u_{yy} \geq 0$ on Ω. Prove that either u is a constant or $u(x) < \sup_\Omega u$ for all $x \in \Omega$.

7. Solve $(3y - 2u)u_x + (u - 3x)u_y = 2x - y$, $u(s, s) = 0$.

Answer any two, each question carries 10 marks, total marks: 20

1. (a) Find a solution of $(1 - x^2)y'' - xy' + p^2y = 0$ using power series method where p is a constant.

   (b) Solve $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ near $x = 0$ where a, b, c are constants and $c \notin \mathbb{Z}$.

2. (a) Let $w(x, t)$ and $v(x, t)$ be solutions of $u_t = u_{xx}$. Find a solution of $u_t = k(u_{xx} + u_{yy})$ such that $u(x, y, 0) = w(x, 0)v(y, 0)$ for $k > 0$.

   (b) Use SV-method to find solutions of $u_{tt} = \Delta u$ on $[0, a] \times [0, b]$ and $t \in [-1, 1]$ satisfying $u(0, y, t) = u(a, y, t) = 0$ and $u(x, 0, t) = u(x, b, t) = 0$.

3. (a) Let D be a bounded open set and u be harmonic on D and continuous on $\overline{D}$. Prove that $\max_{\overline{D}} u = \max_{\partial D} u$.

   (b) Prove that $f \in C^2(\mathbb{R})$ is harmonic on $\mathbb{R}$ if and only if f has mean value property (on $\mathbb{R}$).